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Charge Redistribution between Cyclotron-Resolved Edge States at High Imbalance¶

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We use a quasi-Corbino sample geometry with independent contacts to different edge states in the quantum Hall effect regime to investigate a charge redistribution between cyclotron-split edge states at high imbalance. We also modify the Büttiker formalism by introducing local transport characteristics in it and use this modified Büttiker picture to describe the experimental results. We find that charge transfer between cyclotron-split edge states at high imbalance can be described by a single parameter, which is a transferred between edge states portion of the available for transfer part of the electrochemical potential imbalance. This parameter is found to be independent of the particular sample characteristics, describing fundamental properties of the interedge-state scattering. From the experiment we obtain it in the dependence on the voltage imbalance between edge states and propose a qualitative explanation to the experimental findings. © *2004 MAIK "Nauka/Interperiodica".*

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Just from the beginning of the quantum Hall investigations, it was understood that edge states play a significant role in many transport phenomena in the quantum Hall effect regime [1]. In a quantizing magnetic field the edge potential bends up the energy levels near the sample edges. At the intersections of the energy levels with the Fermi level, edge states are formed. It was a paper of Büttiker [2] that proposed a formalism for the Hall resistance calculation regarding a transport through edge states. This model was further developed by Chklovskii *et al.* [3] for electrostatically interacting electrons. The interaction modifies one-dimensional Büttiker edge states into strips of incompressible electron liquid of finite widths. It was shown theoretically [2] and confirmed in experiments [4] that quantum Hall resistance is not sensitive to the interedge-channel scattering. Nevertheless, the properties of this scattering can be investigated by using the selective edge channel population methods.

Most experiments have been performed in the Hall-bar geometry by using the cross gate technique [4]. These experiments have revealed the interedge-scattering dependence on the magnetic field, temperature and filling factor [4]. In the Hall-bar geometry, the experiments are at low imbalance conditions, when the energy difference between edge states is smaller than the spectral gaps. An attempt to increase the edge states imbalance by closing cross gates dramatically decreases the experimental accuracy, as was mentioned in [5].

Another experimental method is the use of the quasi-Corbino sample geometry [6, 7]. In this geometry two nonconnecting etched edges are formed in the sample. A cross gate is used to redirect some edge states between etched edges and to define an interaction region at one edge. Because the interacting edge states originate from different edges of the sample, they are independently contacted and direct interedge-scattering investigations become possible at any imbalance between edge states. This imbalance is controlled by the applied voltage, and in dependence of its sign, the edge potential profile between edge states becomes stronger or flatter. In the latter case, at some voltage imbalance, the potential barrier between edge states disappears, leading to a steplike behavior of the corresponding branch of the I–V curve. This effect opens a path to use the quasi-Corbino geometry for spectroscopic investigations at the sample edge. Recently, the quasi-Corbino geometry was used to investigate the edge spectrum of single- [7] and double-layer [8] two-dimensional electron structures. It was also understood that, in the transport between spin-resolved edge states at high imbalance (i.e., higher than the spectral gaps), nuclear effects become important [9].

When developed, Büttiker formalism was intended to describe a high accuracy of the sample resistance quantization in the quantum Hall effect regime. For this reason, it depicts the interedge scattering by integral sample characteristics practically as scattering between ohmic contacts. This picture becomes inconvenient

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while describing a charge transfer between edge states at high imbalance, where the scattering by *definition* takes place on small lengths, much smaller than the sample size.

Here, we investigate a charge transfer between cyclotron-split edge states at high imbalance. We modify Büttiker formalism by introducing local transport characteristics in it. We find that charge transfer can be described by a single parameter, which is the transferred portion of the available for the transfer part of the electrochemical potential imbalance. This modified Büttiker picture is used to describe details of charge transfer while current is overflowing between edge channels.

Our samples are fabricated from a molecular beam epitactically-grown GaAs/AlGaAs heterostructure. It contains a two-dimensional electron gas (2DEG) located 70 nm below the surface. The mobility at 4 K is 800000 cm^2/V s, and the carrier density 3.7×10^{11} cm^{-2}. Samples are patterned in a quasi-Corbino geometry [7], see Fig. 1. Rectangular mesa has an etched region inside. Ohmic contacts are made to both (inner and outer) edges of the sample. A Shottky gate is patterned around the inner etched area, leaving an uncovered T-shaped region between inner and outer edges. This region forms a narrow (about several microns) strip of uncovered 2DEG near the outer edge of the sample which is called gate gap. Here, we present data from the sample with 5 μm gate-gap width, while 2-, 10-, and 20-μm gate-gap samples are also investigated, showing identical experimental results.

In our experimental setup, one of the inner contacts is always grounded. In a quantizing magnetic field, at filling factors $\nu = 3, 4$, we deplete 2DEG under the gate to a smaller filling factor $g = 2$, redirecting cyclotron-split ν–g edge states from inner to outer edges of the sample. We apply a dc current to one of the outer contacts and measure a dc voltage drop between two others inner and outer contacts at a temperature of 30 mK. By switching current and voltage contacts I–V, traces for four different contact combinations can be investigated. Because of independent ohmic contacts to the cyclotron-split edge states, the measured voltage U is connected to the voltage drop V between edge states in the gate gap, which is directly the energy shift eV between them. For example, $U = V$ for contact combination at which contacts 4 and 2 are current contacts and *3* and *1* are voltage ones, as denoted in Fig. 1.

Examples of experimental I–V curves are presented in the insets to Figs. 2 and 3 for two groups of cyclotron-split edge states. While increasing the current from zero to positive values, the measured voltage rises abruptly to some value V_{th}. There is practically no current before $V = V_{th}$, but after it, the voltage is a roughly linear function of the current. This linear law is valid for hundreds of nanoAms, see main Figs. 2 and 3, up to our highest applied currents for filling factor combination $\nu = 4$, $g = 2$. For $\nu = 3$, $g = 2$, at high currents, there is a strong deviation from the linear law. The deviation starts from twice the onset voltage $2V_{th}$ and leads to increasing resistance in respect to the linear dependence. It cannot be due to overheating of the sample by

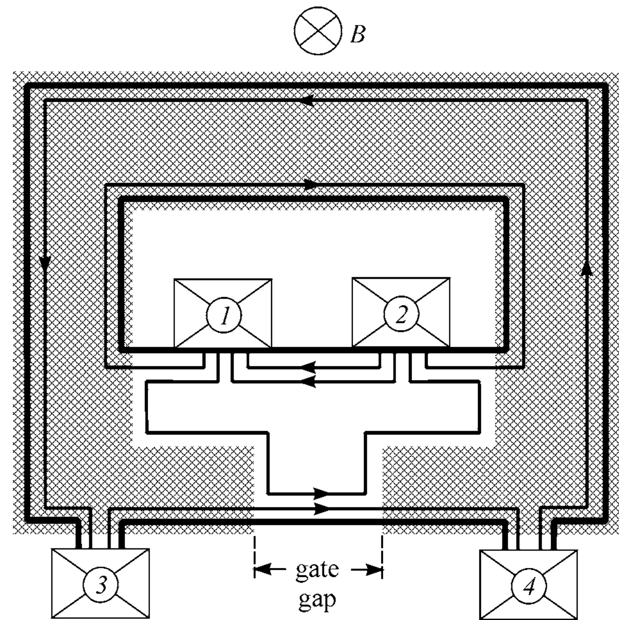

Fig. 1. Schematic diagram of the pseudo-Corbino sample geometry. Contacts are positioned along the etched edges of the ring-shaped mesa (thick outline). The shaded area represents the Schottky gate. Arrows indicate the direction of electron drift in the edge states.

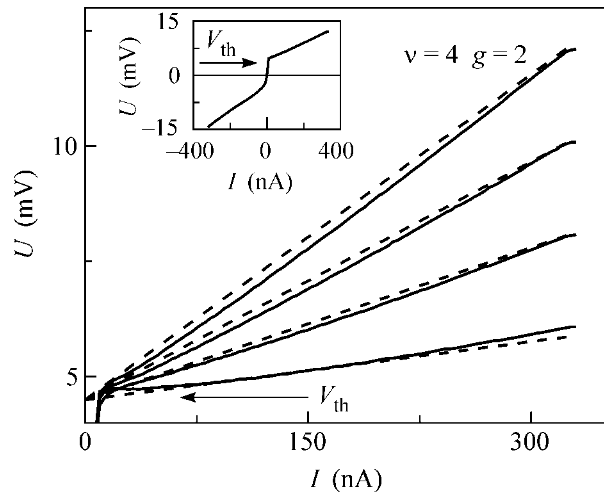

Fig. 2. Positive branches of experimental I–V curves for filling factors $\nu = 4$, $g = 2$ for different contact configurations. They are (from up to down) current (4–2), voltage (3–1); (4–1), (3–2); (3–2), (4–1); (3–1), (4–2) as depicted in Fig. 1. The inset shows an example of the experimental I–V curve in a whole sweeping range with marked threshold position. $V_{th} = 4.5$ mV. The magnetic field is $B = 3.9$ T.

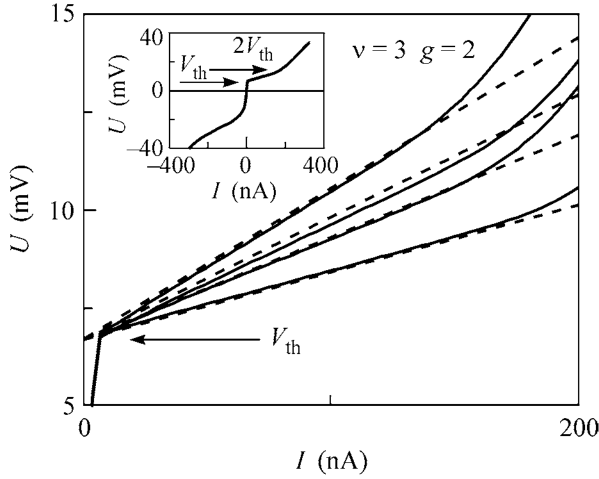

Fig. 3. Positive branches of experimental *I–V* curves for filling factors $\nu = 3$, $g = 2$ for different contact configurations. They are (from up to down) current (4–2), voltage (3–1); (4–1), (3–2); (3–2), (4–1); (3–1), (4–2) as depicted in Fig. 1. The inset shows an example of the experimental *I–V* curve in a whole sweeping range with marked threshold and twice threshold positions. $V_{th} = 6.7$ mV. The magnetic field is $B = 5.1$ T.

the current, because it would diminish the resistance, in contradiction with the experiment, see Fig. 3.

In Figs. 2 and 3, positive *I–V* branches are shown for four different contact combinations. As can be seen from the figure, there is still small nonlinearity of the curves. The behavior described above is valid for all of them and is very reproducible from sample to sample and in cooling cycles. Positive branches start from the same threshold voltage, which is fixed for a given filling factor combination. The threshold voltage values are close to the cyclotron splitting in the corresponding field but smaller by approximately 2 mV, see [7]: $V_{th} = 4.5$ mV for $\nu = 4$, $g = 2$ ($\hbar\omega_c = 6.7$ meV) and $V_{th} = 6.7$ mV for $\nu = 3$, $g = 2$ ($\hbar\omega_c = 8.8$ meV).

While sweeping the current to the negative values, there is no clear defined onset: the voltage is rising with rising the current practically from a zero value. The negative branch of the *I–V* curve is clearly nonlinear for any currents, see insets to Figs. 2 and 3. The exact form of the branch is dependent on the cooling procedure and may vary from cycle to cycle.

To be correct, Büttiker formalism [2] cannot be conveniently applied to transport at high imbalance. It describes *integral* sample resistance, so in the case of nonlinear *I–V* curve, the Büttiker transmission coefficients become dependent on the voltage imbalance between edge states.

As an example, let us consider a filling factor combination $\nu = 4$, $g = 2$. Our sample can be described by the equations [2]

$$\begin{aligned} I_1 &= 4\frac{e}{h}\mu_1 - 4\frac{e}{h}\mu_2, \\ I_2 &= 4\frac{e}{h}\mu_2 - 2\frac{e}{h}\mu_1 - 2\frac{e^2}{h}(T_{21}\mu_1 + T_{23}\mu_3), \\ I_3 &= 2\frac{e}{h}\mu_3 - 2\frac{e}{h}\mu_4, \\ I_4 &= 2\frac{e}{h}\mu_4 - 2\frac{e}{h}(T_{41}\mu_1 + T_{43}\mu_3), \end{aligned} \tag{1}$$

where I_i is the current flowing in the ith contact, i is the electrochemical potential of the ith contact, and $\{T_{ij}\}$ is the matrix of transmission coefficients [2]. These coefficients are not independent: because of the charge conservation in the gate gap, we can write

$$\begin{aligned} T_{21} + T_{41} &= 1, \\ T_{23} + T_{43} &= 1. \end{aligned} \tag{2}$$

Also, from symmetry considerations, we should mention that

$$T_{23} = T_{41}.$$

It means that every transmission coefficient can be expressed through a single value, which we define as $T = T_{23}$.

Let the current flow between contacts 4 and 1, and use contacts 3 and 2 to measure the voltage drop. For these experimental conditions, the flowing current is $I_{41} = I_1 = -I_4$ and there is no current in the voltage probes $I_2 = I_3 = 0$. Also, the voltage drop is the difference of the electrochemical potentials of the potential contacts, so $eU_{32} = \mu_3 - \mu_2$. By solving system (1) with relations (2) and herein, we can obtain

$$U_{32} = \frac{2-T}{4T}\frac{h}{e^2}I_{41}. \tag{3}$$

Relation (3) can be used to calculate T from the experimental *I–V* trace, see Fig. 4. The dependence $T(V)$ is strongly nonlinear. It starts from the threshold voltage, because below threshold practically no current is flowing, so the transmission T is practically zero. While the voltage imbalance between edge states V increases, $T(V)$ is monotonically rising and asymptotically tends to the equilibrium Büttiker value $T = 1/2$ at high voltages V. $T(V)$ dependence has a universal character: when obtained, it can be used to describe the experimental *I–V* traces for any given contact combination at fixed filling factors. One should calculate the current–voltage relation for this contact combination from Eq. (1) and introduce the above $T(V)$ into it to

obtain the experimental I–V curve. We will demonstrate this fact below in a physically more transparent manner.

Using strongly nonlinear transparency $T(V)$ is too sophisticated to analyze the overflowing current at $V > V_{th}$; e.g., it is not clear the physical origin of the linear regions on the experimental I–V curves. For nonlinear transport in the gate gap, it is obvious to introduce local transport characteristics instead of the integral Büttiker transmission coefficient T. From the positive branch of the experimental I–V curve, we conclude that there is practically no current between edge states below the threshold voltage. In the Büttiker picture of edge states, it means that both edge states are injecting and leaving the gate-gap region with their own electrochemical potentials μ_1 and μ_3, originating from corresponding ohmic contacts 1 and 3. Currents flowing in the gate gap are equal to $e/h\mu_1$ and $e/h\mu_3$ in the inner and outer edge states, respectively. A current *between* edge states starts to flow; then, the difference in electrochemical potentials exceeds the threshold voltage. In other words, only some part of the incoming electrochemical imbalance $(\mu_3 - \mu_1 - eV_{th})$ is available for redistribution between edge states. It is obvious in this case to describe the current between edge states as $\alpha(\mu_3 - \mu_1 - eV_{th})e/h$, where α is a parameter describing a portion of the available part of electrochemical potential imbalance, which is in fact transferred between edge states. For the above filling factor combination $\nu = 4$, $g = 2$, it is clear that $\alpha = 1/2$ means equal redistribution between edge states. The edge states are leaving the gate-gap region with mixed electrochemical potentials $\mu_1 + \alpha(\mu_3 - \mu_1 - eV_{th})$ and $\mu_3 - \alpha(\mu_3 - \mu_1 - eV_{th})$. By introducing these values into Büttiker formulas (1), we have the following equations instead of (1)–(3):

$$I_1 = 4\frac{e}{h}\mu_1 - 4\frac{e}{h}\mu_2,$$

$$I_2 = 4\frac{e}{h}\mu_2 - 2\frac{e}{h}\mu_1 - 2\frac{e}{h}(\mu_1 + \alpha(\mu_3 - \mu_1 - eV_{th})),$$

$$I_3 = 2\frac{e}{h}\mu_3 - 2\frac{e}{h}\mu_4, \qquad (4)$$

$$I_4 = 2\frac{e}{h}\mu_4 - 2\frac{e}{h}(\mu_3 - \alpha(\mu_3 - \mu_1 - eV_{th})).$$

In this case there is no need for any additional relations (all the necessary information is indeed in equations (4)) and the only parameter α has a clear physical sense: it is a transferred between edge states portion of the available for the transfer part of the electrochemical potential imbalance between edge states. The aforementioned combination of filling factors and contacts can be described by I–V relation

$$U_{32} - V_{th} = \frac{2-\alpha}{4\alpha}\frac{h}{e^2}I_{41}. \qquad (5)$$

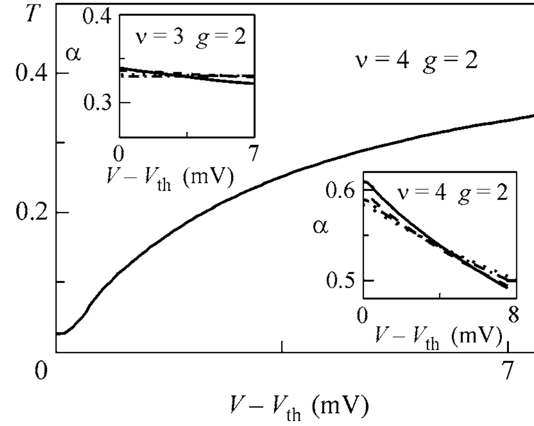

Fig. 4. The dependence of the Büttiker transmission coefficient T on the voltage imbalance between cyclotron-resolved edge states at filling factors $\nu = 4$, $g = 2$, starting from the threshold voltage. Insets show the dependences of the transport parameter α (see text) as obtained from I–V curves at four different contact configurations for $\nu = 3$, $g = 2$ and $\nu = 4$, $g = 2$ filling factors, respectively.

It is important to mention that, because α is the local characteristic of the interedge state transport, it should be independent from the contact combination. In other words, a single value of α obtained from different I–V curves is a test of the consistency of our description.

The linear behavior of experimental I–V curves after the threshold means a constant slope in Eq. (5) and, therefore, a constant α. In Figs. 2 and 3, the linear regions of experimental I–V curves are fitted by dashed lines. These lines are calculated from formulas like Eq. (5) with constant single a for every filling factor combination. The used values of α are 0.55 for $\nu = 4$, $g = 2$ factors and 0.34 for $\nu = 3$, $g = 2$. It can be seen from the figures that dashed lines fit the experimental curves quite well, even in view of small nonlinearity of the experimental curves. The same values of α were obtained from similar linear fits for other samples with different gate-gap widths. We should conclude that α depends only on the filling factor combination and, therefore, describes fundamental properties of the interedge-state transport.

The fact that the experimental traces are not exactly linear, see Figs. 2 and 3, indicates that there is a slow dependence of α on the voltage imbalance between edge states. Using formulas like Eq. (5), it is possible to extract this dependence of α directly from the experimental traces. In the insets to Fig. 4, the dependence of α is depicted as a function of the voltage imbalance V between edge states for two different filling factor combinations. Just from the definition, α is zero before the threshold, it jumps to values that are close but slightly higher than ones for full equilibration between all involved edge states ($\alpha_{eq} = 1/2$ for $\nu = 4$, $g = 2$ filling factors and $\alpha_{eq} = 1/3$ for $\nu = 3$, $g = 2$) and then slowly

diminishes with increasing voltage imbalance V between edge states. For a single filling factor combination, $\alpha(V)$ traces obtained from different contact configurations deviate within 2%, which is of the order of our experimental accuracy, which also indicates the universal character of the α parameter.

Let us discuss the obtained dependence of α on voltage imbalance between edge states, see insets to Fig. 4. It is important to mention that α by definition describes the *resulting* mixing of the electrochemical potentials, while charge transfer takes place on the whole length of the gate-gap width. This charge transfer changes the electrochemical potentials of the edge states. It means that, while at one (injection) corner of the gate gap, the energy shift between edge states equals the depicted in the figures voltage imbalance V between them, the edge potential profile between edge states is flattening while moving away from the injection corner. At some point the edge profile becomes flat. If this point is really within the gate gap, full equilibration between edge states is established, and there should be no further charge transfer on the rest of the gate-gap width. The resulting value of α in this case can be expected to be exactly equal to the equilibrium one. The experimental fact that the values of α are higher than the equilibrium ones indicates that charge transfer *in the same direction* is still taking place even after the equilibration point. In this case the slow dependence of α on the voltage imbalance V becomes clear: at higher imbalance V, a higher amount of electrons should be transferred between edge state to flatten the potential; thus, the point of equilibration moves to the opposite to the injection corner of the gate gap and the "length of overflowing" (on which an additional charge is transferred) becomes shorter. After leaving the gate gap, equilibration is not established at all, so α becomes smaller than the equilibrium value. This behavior can be clearly seen in insets to Fig. 4. The origin of the "overflowing" behavior is still unclear and requires further theoretical investigations. One qualitative explanation can be proposed here: the value of the threshold voltage V_{th} is determined by the cyclotron splitting but not exactly, see [7]. At least the energy level broadening has an influence on the value of V_{th}, and maybe any other factors. In this case we can suppose a small variation of V_{th} along the gate gap, which leads to the additional charge transfer.

It is worth mentioning that, for the filling factor combination $\nu = 3$, $g = 2$, experimental values of α vary around the value 1/3. This is the equilibrated value at which all three edge states are involved in the charge transfer. It means that electrons from inner edge state having spin in the field direction, "up," are moving both in the neighbor edge state with spin "down" and in the outer edge state with spin "up." These processes should go together: without high voltage imbalance, equilibration between spin-split edge states goes on a millimeter distance [5]; so, to have the full equilibration between all three edge states on few microns, as well process with spin flip should be present as one without it. (For $\nu = 4$, $g = 2$ filling factors, where the transport goes between two pairs of equilibrated spin-split edge states, spin flip is not needed.) At voltages above V_{th} but below $2V_{th}$, electrons are moving by vertical relaxation through the cyclotron gap and a diffusion in space afterwards. In the relaxation process the energy is changing by emitting a photon (in spin-flip transfer) or a phonon (without spin-flip). As the voltage imbalance exceeds $2V_{th}$, the energy levels are bent enough to allow *horizontal* transitions between edge states [9]. In these transitions, electron spin is flipping due to flopping of nuclear spin, in so called flip–flop processes, which leads to the formation of a nuclear polarized region in the gate gap. This process is well known in the literature [9–11] as a dynamic nuclear polarization. Once appeared, a region of dynamically polarized nuclei influences the electron energies through the effective Overhauser field. Overhauser field is effectively compensating the external field for the Zeeman splitting and can be in GaAs as high as 5 T, see [12]. Thus, it can significantly change the space distance between spin-split edge states and, therefore, increase the distance for the charge transfer in the gate gap (which is determined by the difference between cyclotron and spin splittings). This gives rise to increase of the resistance, once makes harder the charge transfer. In the experiment, it is at this voltage $V = 2V_{th}$ the experimental I–V traces change their slopes for $\nu = 3$, $g = 2$ filling factors, see the inset to Fig. 3. A hysteresis on the I–V curves for $\nu = 3$, $g = 2$ above the voltage $2V_{th}$ is also present (not shown in the figure), which is a key feature of the dynamic nuclear polarization [9–11].

We used a quasi-Corbino sample geometry with independent contacts to different edge states in the quantum Hall effect regime to investigate a charge transfer between cyclotron-split edge states at high imbalance. We found that charge transfer between cyclotron-split edge states at high imbalance can be described by a single parameter, which is the transferred portion of the available for transfer part of the electrochemical potential imbalance between edge states. From the experiment we obtained this parameter in its dependence on the voltage imbalance between edge states and proposed a qualitative explanation.

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